Use of overlapping finite elements for connecting arbitrary surfaces with dual formulations

H. Zaidi, L. Santandrea, G. Krebs and Y. Le Bihan
Laboratoire de Génie Electrique de Paris (LGEP), CNRS UMR 8507, Supelec, UPMC-P6, Univ. Paris-Sud 11
11 rue Joliot Curie, 91192 Gif-Sur-Yvette cedex France.
Guillaume.krebs@lgep.supelec.fr

Abstract — In this paper, a method for the connection of non-conform arbitrary surfaces by overlapping finite element method is presented. Both scalar and vector degrees of freedom can be considered. The use of reference elements allows to simplify the implementation of the method.

I. INTRODUCTION

The taking into account of the movement in the finite element (FE) modeling is an essential point. For the electrical devices such as motors or sensors, typical features can be then obtained from movement (emf, torques, impedances…). Several methods have been developed and used [1]-[2]. Most of them permit to connect meshed surfaces in contact. As long as the airgap between the fixed and moving part is constant, these method behave correctly [3] with quite reasonable computation times.

In several problems, the distance between the parts to connect can change over time and space (rotor eccentricity, flexible sensor…). Thus the previous methods can not be used anymore. A complete or partial remeshing is not acceptable when an hundred of sensor positions are required for example. One solution is to use overlapping finite elements [4]. Two surfaces separated by a non-meshed volume area, see Fig.1, can be connected using this method.

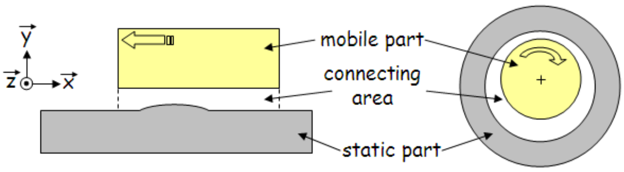

Fig. 1. Examples of configuration that can be solved using overlapping finite elements (translating or rotating movement).

Recent contributions have been proposed with this method. In [5], the overlapping elements are used within a vector potential formulation but with a constant volume for the connecting area. In [6], a use of the reference elements has been proposed for a scalar potential formulation with a non-constant airgap area. In this paper, an extension of the previous works is proposed. The subject is related to the connection of arbitrary surfaces in the case of a vector potential formulation with overlapping finite elements. For this purpose a reference element is implemented.

II. OVERLAPPING FINITE ELEMENTS

The principle of the method has been described in previous works, so only a brief recall is done here. It consists in a projection of the nodes of one boundary surface to the other one and reversely creating then elements that overlap (Fig. 2). Theses elements are used to define the shape functions associated to the unknowns. The airgap between the surfaces to connect is not constant, so the overlapping elements are unconventional (the element created by the projections of N_4 and N_3 is not a trapeze).

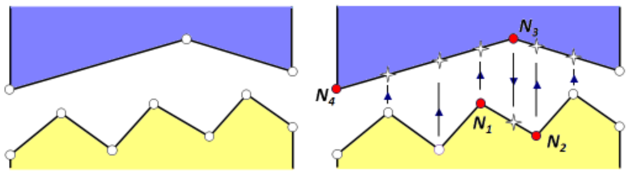

Fig. 2: Node projection when the surfaces to connect are not planar.

The connection of the two meshed domains is performed via the calculation of integral terms located in the overlapping domain. For this purpose, special edge functions for the unconventional elements could be defined and directly used for the calculation of **curl** terms. However such an approach is not obvious and can lead to significant computation times. Using [5] and [6], a new use of the reference element for the calculation of edge shape function in unconventional overlapping elements is proposed in what follows.

III. CALCULATION OF EDGE SHAPE FUNCTIONS IN UNCONVENTIONAL OVERLAPPING ELEMENTS

For the sake of clarity, the used FE formulation will be detailed in the extended version. The unknowns are associated with edge elements so edge shape functions have to be calculated. Let us consider the example of Fig.3 where the nodes N_1, N_2 and N_3 are projected and thereby creates an unconventional overlapping element.

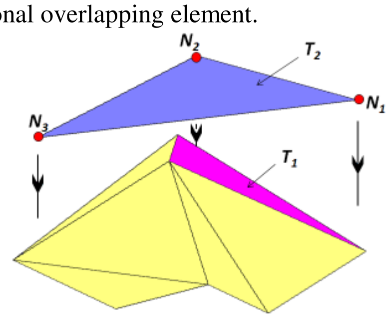

Fig. 3. Example of projection leading to unconventional overlapping elements.

To calculate the integral terms, the overlapping area is subdivided into prisms. For example the prism P_i of Fig.4 comes from the projection of the triangles T_1 and T_2.

The principle of the proposed approach is to link the nodes and edges of the reference element to the nodes and edges of the current arbitrary prism (transfer of the set of nodes and edges of the reference element to the set of nodes and edges of T_1 and T_2).

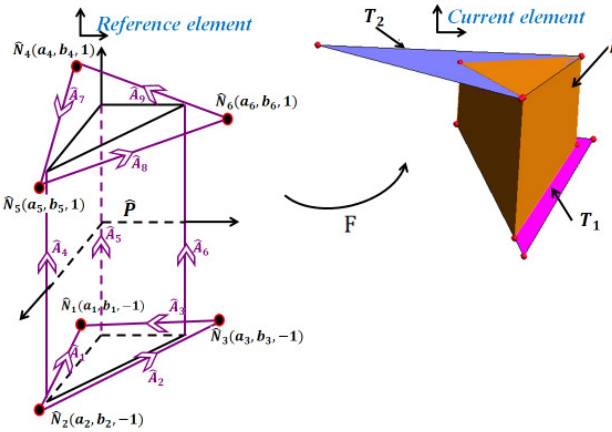

Fig. 4. Scheme for the calculation of shape functions.

The prism $\hat{P}$ is the reference element of P_i. The expression of the variables (a_i, b_i) $i \in \{1,...,6\}$ will be given in the extended paper. F is the application of the transformation defined by:

$$F : \hat{P} \rightarrow P_i \text{ such as } F(x, y, z) =< \hat{N}(x, y, z) > .T \tag{1}$$

with:

$$\begin{aligned} < \hat{N}(x, y, z) > =< & \hat{\lambda}_1(x, y, z), \hat{\lambda}_2(x, y, z), \hat{\lambda}_3(x, y, z) \\ & , \hat{\lambda}_4(x, y, z), \hat{\lambda}_5(x, y, z), \hat{\lambda}_6(x, y, z) > \end{aligned} \tag{2}$$

the nodal shape functions of the six nodes of the reference element and $T = \{x_k \;\; y_k \;\; z_k\}_{1 \le k \le 6}$ containing the coordinates of the six points that construct the two triangles T_1 and T_2. The expressions of the edge functions are given by:

$$\begin{cases} \hat{\psi}_i(x, y, z) = \dfrac{1-z}{2} \hat{\psi}_i^{\hat{t}1}(x, y) \; i \in \{1,2,3\} \\ \hat{\psi}_i(x, y, z) = \hat{\psi}_i^{\hat{P}}(x, y, z) \; i \in \{4,5,6\} \\ \hat{\psi}_i(x, y, z) = \dfrac{1+z}{2} \hat{\psi}_i^{\hat{t}2}(x, y) \; i \in \{7,8,9\} \end{cases} \tag{3}$$

The variables $\hat{\psi}_i^{\hat{t}1}$ $i \in \{1,2,3\}$ are the three edge shape functions associated to the triangle $\hat{t}_1$ formed by three nodes $\hat{N}_1$, $\hat{N}_2$ and $\hat{N}_3$ (Fig.4). $\hat{\psi}_i^{\hat{t}2}$ $i \in \{7,8,9\}$ are the three edge shape functions associated to the triangle $\hat{t}_2$ formed by the three nodes $\hat{N}_4$, $\hat{N}_5$ and $\hat{N}_6$. Finally, $\hat{\psi}_i^{\hat{P}}$ $i \in \{4,5,6\}$ are the three edge shape functions associated to the vertical edge of prism $\hat{P}$.

In order to verify the properties of the edge shape functions the following figure shows the distribution of the shape function of the edge (L) of a triangle (top) projected on a non-planar surface constituted of a set of triangles (bottom).

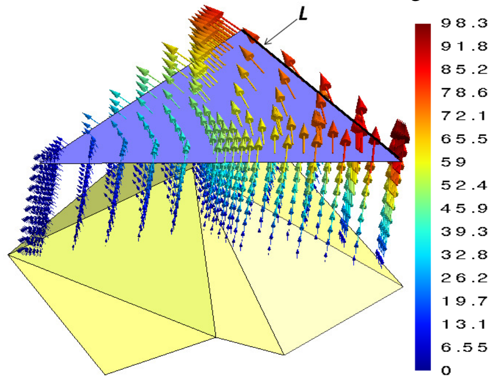

Fig. 5. Edge shape function associated with the edge L.

IV. Test and Result

Fig. 6 shows the test case study, which consists in a cube divided into two by a curved surface with an applied vector potential difference $\Delta \boldsymbol{a}$ equal to 2**L* (*L*: height of the box). The local zoom of the results presented at the right of the figure shows that the B-field distribution is correctly calculated with regards to its direction and intensity.

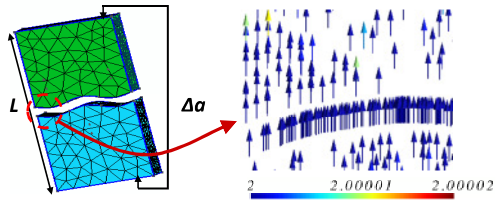

Fig. 6. Test case.

V. Acknowledgements

This work was supported by DIGITEO, Île-de-France Region and CEA LIST.

VI. References

[1] C. W. Trowbridge and J. K. Sykulski, “Some Key Developments in Computational Electromagnetics and Their Attribution”, *IEEE trans. on Magn.*, vol. 42, no.4,pp.503-508, April 2006.

[2] M. Racheka, M. Feliachi, "3-D movement simulation techniques using FE methods: application to eddy current non-destructive testing", *NDT&E International*, pp.35-42, September 2006.

[3] O. J. Antunes, J. P. A. Bastos, N. Sadowski, A. Razek, L. Santadrea, F. Bouillault, and F. Rapetti, “Comparison between nonconforming movements methods”, *IEEE Trans. on Magnetics*, vol. 42, n°4, 2006.

[4] I.A. Tsukerman, "Overlapping finite element for problems with movement", *IEEE Trans.on Magnetics*, vol. 28, no. 5, September 1992.

[5] G. Krebs, T. Henneron, S. Clénet, Y. Le Bihan, “Overlapping finite elements used to connect non-conforming meshes in 3D with a vector potential formulation”, *IEEE Conference on Electromagnetic Field Computation (CEFC)*, Chicago(USA), May 2010.

[6] H. Zaidi, L. Santandrea, G. Krebs, Y.Le Bihan, “A contribution to connect non-conform meshes with overlapping finite elements”, *International Symposium on Numerical Field Calculation in Electrical Engineering (IGTE)*, Graatz (Austria), September 2010.